

[2] SUPPOSE  $f(3)=17$

SINCE  $f$  IS DIFFERENTIABLE ON  $[-1, 3]$ ,  $\frac{1}{2}$

THEREFORE  $f$  IS CONTINUOUS ON  $[-1, 3]$ ,  $\frac{1}{2}$

AND BY MVT, FOR SOME  $c \in (-1, 3)$   $\frac{1}{2}$

$$\frac{1}{2} \quad \left[ f'(c) = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{17 - (-2)}{4} = \frac{19}{4} < 5 \right] \textcircled{1}$$

BUT  $f'(x) > 5$  ON  $[-1, 3]$  (CONTRADICTION)

SO, IT IS NOT POSSIBLE THAT  $f(3)=17$

$$[3][a] \lim_{x \rightarrow \frac{\pi}{2}} (1-2\cos x)^{\tan x} = \lim_{x \rightarrow \frac{\pi}{2}} e^{\tan x \ln(1-2\cos x)}$$

$$(1-0)^{\infty} \rightarrow 1^{\infty}$$

or  $(1-0)^{-\infty} \rightarrow 1^{-\infty} \rightarrow \frac{1}{\infty}$

$$\stackrel{(1)}{=} e^{\lim_{x \rightarrow \frac{\pi}{2}} \tan x \ln(1-2\cos x)} = e^{-2} \quad (1)$$

$$\stackrel{(1)}{=} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(1-2\cos x)}{\cot x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{1}{1-2\cos x} \cdot 2\sin x}{-csc^2 x} = \lim_{x \rightarrow \frac{\pi}{2}} - \frac{2\sin^3 x}{1-2\cos x} \quad (2)$$
$$= \frac{-2(1)^3}{1-0} = -2 \quad (\frac{1}{2})$$

$$[6] \quad \lim_{x \rightarrow \infty} \frac{7+2x}{\sqrt[3]{5-4x^3}} \cdot \frac{|x|}{|x|}$$

$\frac{1}{2}$

$$= \lim_{x \rightarrow \infty} \frac{\frac{7}{x} + 2}{\sqrt[3]{\frac{5}{x^3} - 4}}$$

$\frac{1}{2}$

$$= \frac{0+2}{\sqrt[3]{0-4}} = -\frac{2}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$$

$$= -\sqrt[3]{2} \quad \frac{1}{2}$$

[4] YES,  $f$  IS CONTINUOUS ON  $(0, \infty)$  WHICH INCLUDES  $[e^{-1}, e^2]$

$$f'(x) = \frac{(2 \ln x) \frac{1}{x} x^3 - (\ln x)^2 3x^2}{(x^3)^2} = \frac{2 \ln x - 3(\ln x)^2}{x^4} \text{ EXISTS ON } [e^{-1}, e^2]$$

$$f'(x) = \frac{(\ln x)(2 - 3 \ln x)}{x^4} = 0 \text{ IF } \ln x = 0 \text{ OR } 2 - 3 \ln x = 0$$

IE.  $x = 1$  OR  $x = e^{\frac{2}{3}}$  BOTH IN  $[e^{-1}, e^2]$

$$f(e^{-1}) = \frac{(-1)^2}{(e^{-1})^3} = \underline{e^3} > 1$$

$$f(1) = \frac{0^2}{1^3} = \underline{0}$$

$$f(e^{\frac{2}{3}}) = \frac{(\frac{2}{3})^2}{(e^{\frac{2}{3}})^3} = \frac{4}{9e^2} < 1$$

$$f(e^2) = \frac{(2)^2}{(e^2)^3} = \frac{4}{e^6} < 1$$

GLOBAL MAX

AT  $(e^{-1}, e^3)$

[5] BQ IS RIGHT.  $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$  so  $f$  IS NOT DIFFERENTIABLE

AT  $x=0 \in (-1, 8)$ . SO MVT DOES NOT APPLY

1/2

[6][a]  $f(x) = 14x^{\frac{10}{3}} + 40x^{\frac{7}{3}} + 35x^{\frac{4}{3}}$     DOMAIN =  $(-\infty, \infty)$   $\textcircled{\frac{1}{2}}$

$f'(x) = \frac{140}{3}x^{\frac{7}{3}} + \frac{280}{3}x^{\frac{4}{3}} + \frac{140}{3}x^{\frac{1}{3}} = \frac{140}{3}(x^{\frac{7}{3}} + 2x^{\frac{4}{3}} + x^{\frac{1}{3}})$

EXISTS ON  $(-\infty, \infty)$

$f'(x) = \frac{140}{3}x^{\frac{1}{3}}(x^2 + 2x + 1) = \frac{140}{3}x^{\frac{1}{3}}(x+1)^2 = 0$

IF  $x^{\frac{1}{3}} = 0$  OR  $(x+1)^2 = 0$

IE. IF  $\textcircled{\frac{1}{3}}x = 0$  OR  $\textcircled{\frac{1}{3}}x = -1$   $\textcircled{\frac{1}{2}}$

$$[b] f''(x) = \frac{140}{3} \left( \frac{7}{3} x^{\frac{4}{3}} + \frac{8}{3} x^{\frac{1}{3}} + \frac{1}{3} x^{-\frac{2}{3}} \right) = \frac{140}{9} x^{-\frac{2}{3}} (7x^2 + 8x + 1)$$

$f'(0) = 0$  BUT  $f''(0)$  DNE  $\rightarrow$  2<sup>ND</sup> DERIVATIVE TEST  
TELLS US NOTHING

$f'(-1) = 0$  BUT  $f''(-1) = 0$   $\rightarrow$  2<sup>ND</sup> DERIVATIVE TEST  
TELLS US NOTHING

OR 0 POINTS IF  
YOU SAID  
"NOT AN  
EXTREMA"

[c]

$$\frac{140}{3} x^{\frac{1}{3}}$$

-

-

+

(1)

$$(x+1)^2$$

+

+

+

(1)

$f'$



-



0



-

-

+

(1)

$f$  HAS A LOCAL MIN AT  $x = 0$  (1/2)

$f$  HAS NO LOCAL EXTREMA AT  $x = -1$  (1/2)



[d]  $f''(x) = \frac{140}{9} x^{-\frac{2}{3}} (7x+1)(x+1)$  DNE AT  $x=0$  ①

①

$= 0$  AT  $x = -1, -\frac{1}{7}$

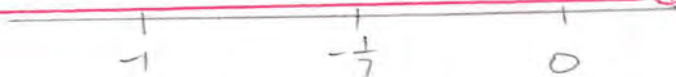
①/2

①/2

$\frac{140}{9} x^{-\frac{2}{3}}$  + + + + ①

$7x+1$  - - + + ①

$x+1$  - + + + ①



$f'''$  + - + + ①

$f$  HAS INFLECTION POINTS AT  $x = -1$  AND  $x = -\frac{1}{7}$  ①/2